Primality of Repunit
An absurd hypothesis: The number of symbols

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Abstract

There is a common feature that characterizes the primes repunit. In this article I show this property, hoping that it is wrong.

Introduction

A repunit is a number consisting of copies of the single digit 1. Examples of repunit are 11, 1111, 11111. A repunit prime is a repunit that is also a prime number.

In base 10, repunits have the form: $\frac{(10^{p}-1)}{9}$ where p is a prime number.

I start my article, saying that the number of digits of a repunit prime number, in base 10, must be odd. We all know that.
We know that p, in Rp, must not only be odd, but must also be a prime number.
In particular, the number of digits "1" in a prime repunit, in base 10, must always be odd. In fact:

Repunit	Decimal expansion	Number of digit "1"	Is it Odd?
R_{19}	1111111111111111111	There are 19 "1"	19 is odd
R_{23}	11111111111111111111111	There are 23 "1"	23 is odd
R_{317}	1111111111... and other 307 "1"	There are 317 "1"	317 is odd
R_{1031}	1111111111... and other 1021 "1"	There are 1031 "1"	1031 is odd
R_{49081}	1111111111... and other 49071 "1"	There are 49081 "1"	49081 is odd
R_{86453}	1111111111... and other 86443 "1"	There are 86453 "1"	86453 is odd
R_{109297}	1111111111... and other 109287 "1"	There are 109297 "1"	109297 is odd
R_{270343}	1111111111... and other 270333 "1"	There are 270343 "1"	270343 is odd

The first requirement of Rp to be prime, in base 10, is that p must be odd (except for R2).

Conjecture

So, I wanted to see if, in other bases, there are other properties. I made many tests and experiments and I discovered the following:

a Repunit number is a prime number or probable prime number IF:

The number of its symbols '3', in base 4, are **even**, AND
The number of its symbols '0', in base 16, are **even**, AND
The number of its symbols '56', in base 64, are **even**, AND
The number of its symbols '7', in base 128, are **even**, AND
The number of its symbols '121', in base 128, are **even**, AND
The number of its symbols '39', in base 256, are **even**, AND
The number of its symbols '255', in base 256, are **even**, AND
The number of its symbols '36', in base 512, are **even**, AND
The number of its symbols '60', in base 512, are **even**, AND
The number of its symbols '74', in base 512, are **even**, AND
The number of its symbols '76', in base 512, are **even**, AND
The number of its symbols '151', in base 512, are **even**, AND
The number of its symbols '182', in base 512, are **even**, AND
The number of its symbols '263', in base 512, are **even**, AND
The number of its symbols '306', in base 512, are **even**, AND
The number of its symbols '309', in base 512, are **even**, AND
The number of its symbols '328', in base 512, are **even**, AND
The number of its symbols '358', in base 512, are **even**, AND
The number of its symbols '361', in base 512, are **even**, AND
The number of its symbols '366', in base 512, are **even**, AND
The number of its symbols '404', in base 512, are **even**, AND
The number of its symbols '433', in base 512, are **even**, AND
The number of its symbols '434', in base 512, are **even**, AND
The number of its symbols '439', in base 512, are **even**, AND
The number of its symbols '495', in base 512, are **even**, AND
The number of its symbols '503', in base 512, are **even**.

Note that the quantity 0 is considered as even.

Examples

For R_{19}, the number of symbols '3', in base 4, is 10 (even).
Indeed R_{19}, in base 4, is 331223131122230223301013013013.

For R_{23}, the number of symbols '3', in base 4, is 6 (even).
Indeed R_{23}, in base 4, is 2112211112210123211312221213013013013.

For R_{317}, the number of symbols '56', in base 64, is 2 (even).
Indeed R_{317}, in base 64, is {58, 60, 27, 61, 50, 57, 5, 63, 58, 18, 48, 49, 24, 50, 14, 63, 24, 20, 62, 48, 22, 52, 12, 59, 51, 28, 50, 10, 28, 17, 54, 45, 17, 45, 33, 61, 20, 52, 48, 56, 4, 54, 11, 29, 44, 8, 26, 45, 61, 21, 33, 43, 63, 46, 47, 33, 45, 21, 11, 12, 15, 2, 0, 7, 19, 49, 6, 39, 57, 3, 10, 12, 42, 48, 34, 55, 36, 46, 13, 25, 54, 10, 10, 27, 56, 49, 58, 63, 40, 19, 58, 5, 59, 24, 38, 0, 10, 27, 30, 1, 19, 41, 62, 40, 13, 21, 30, 11, 57, 3, 35, 1, 18, 17, 39, 42, 59, 59, 52, 55, 31, 61, 39, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7}.

For R_{1031}, the number of symbols '74', in base 512, is 2 (even).
Indeed R_{1031}, in base 512, is {3, 171, 420, 109, 165, 270, 388, 218, 339, 295, 187, 327, 510, 277, 344, 7, 478, 438, 74, 241, 352, 475, 86, 61, 24, 148, 348, 360, 506, 268, 57, 496, 345, 54, 232, 24, 155, 251, 79, 107, 264, 394, 333, 257, 232, 41, 494, 65, 277, 95, 269, 399, 369, 30, 334, 465, 159, 166, 443, 448, 33, 290, 43, 194, 46, 278, 40, 56, 487, 230, 311, 186, 243, 218, 379, 479, 23, 185, 162, 172, 39, 308, 315, 250, 292, 473, 218, 130, 470, 231, 28, 263, 285, 437, 345, 488, 432, 226, 350, 415, 260, 143, 407, 359, 477, 411, 420, 32, 31, 252, 393, 143, 362, 259, 126, 419, 94, 381, 165, 466, 414, 376, 171, 374, 39, 295, 47, 451, 57, 122, 134, 113, 332, 99, 18, 6, 352, 259, 253, 324, 290, 21, 417, 40, 352, 476, 334, 78, 104, 232, 58, 472, 98, 387, 37, 510, 493, 357, 80, 233, 319, 198, 244, 416, 353, 380, 85, 274, 174, 476, 121, 320, 95, 315, 327, 167, 373, 53, 484, 509, 485, 461, 113, 112, 251, 142, 161, 415, 122, 240, 63, 41, 124, 329, 263, 97, 456, 441, 191, 235, 185, 336, 354, 79, 426, 166, 367, 411, 134, 25, 363, 109, 11, 481, 232, 258, 97, 3, 245, 498, 75, 118, 442, 399, 216, 213, 24, 298, 57, 140, 85, 162, 84, 42, 210, 464, 343, 73, 142, 438, 423, 267, 152, 177, 74, 473, 412, 501, 268, 63, 88, 66, 430, 349, 11, 314, 184, 256, 385, 367, 508, 502, 2, 14, 227, 457, 103, 56, 455}

And so on.

Results

I have implemented an algorithm to calculate and verify the conjecture. The results are as follows:

- Candidate=R_3 (exception)
- Candidate=R_5 (exception)
- Candidate=R_{11} (exception)
- Candidate=R_{19}
- Candidate=R_{23}
- Candidate=R_{107} (exception)
- Candidate=R_{317}
- Candidate=R_{1031}
- Candidate=R_{49081}
- Candidate=R_{86453}
- Candidate=R_{109297}
- Candidate=R_{270343}

The only exceptions are R_3, R_5, R_{11}, R_{107} that validate the conjecture, but they are composite numbers. It's useful to perform a small factorization to find any factors.

Current tests

Currently I have tested my conjecture for exponents up to

p=62,000,000

and there are NO other candidate repunit prime numbers (R_p).

Thanks to Danilo Nitsche for helping me in research.

Conclusions

At moment I'm trying the conjecture with greatest exponents, and I hope that it is wrong. Excuse my English, not very good, but I'm Italian.

Good luck to all.

Giovanni Di Maria
Caltanissetta, Jan 19, 2011

Appendix A

Mathematica source code

```
(*-----Funzione delle quantità di simboli in altre basi------*)
Test[r_]:=Block[{colpi},
   colpi=0;
   If[!EvenQ[DigitCount[r,4,3]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,16,0]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,64,56]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,128,7]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,128,121]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,256,39]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,256,255]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,36]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,60]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,74]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,76]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,151]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,182]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,263]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,306]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,309]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,328]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,358]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,361]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,366]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,404]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,433]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,434]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,439]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,495]],Return[colpi],colpi++];
   If[!EvenQ[DigitCount[r,512,503]],Return[colpi],colpi++];
   Return[colpi];
]

(*-----Ci sono fattori Piccoli?------*)
FattoriPiccoli[r_,k_]:=Block[{i,risultato,fattore,LimiteFattoriPiccoli},
risultato=0;
LimiteFattoriPiccoli=100;
For[i=1,i<=LimiteFattoriPiccoli,i++,
   fattore=(i*2)*k+1;
   If[PrimeQ[fattore] && Mod[r,fattore]==0,
      risultato=1;
      Break[]];
   ];
   Return[risultato];
]

(*-----SIEVING------*)
tot=0;
colpimax=0;
Monitor[
   For[k=1,k<=40000000,k++,
      If[Mod[k,10000]==0,Print["Saved at k=",k," at ",DateString[]];NotebookSave[]];
      If[!PrimeQ[k],Continue[]];
      tot++;
      ru=(10^k-1)/9;
      colpi=0;
      If[ FattoriPiccoli[ru,k]==0,colpi=Test[ru]];
      If[colpi>=colpimax,
         colpimax=colpi;
         out01="A Good Candidate is ";
         out02=StringJoin["R(",ToString[k],")"];
         out02=Style[out02,{Red,Bold}];
         out03=StringJoin["     Shots = ",ToString[colpi]];
         Print[out01,out02,out03];
      ];
   ];
,k]
```

Appendix B

Program results as screenshot

```
A Good Candidate is R(2)     Shots = 0
A Good Candidate is R(3)     Shots = 0
A Good Candidate is R(5)     Shots = 0
A Good Candidate is R(7)     Shots = 0
A Good Candidate is R(11)     Shots = 26
A Good Candidate is R(19)     Shots = 26
A Good Candidate is R(23)     Shots = 26
A Good Candidate is R(317)     Shots = 26
A Good Candidate is R(1031)     Shots = 26
Saved at k=10000 at Thu 20 Jan 2011 11:09:11
Saved at k=20000 at Thu 20 Jan 2011 11:09:13
Saved at k=30000 at Thu 20 Jan 2011 11:09:16
Saved at k=40000 at Thu 20 Jan 2011 11:09:20
A Good Candidate is R(49081)     Shots = 26
Saved at k=50000 at Thu 20 Jan 2011 11:09:24
Saved at k=60000 at Thu 20 Jan 2011 11:09:29
Saved at k=70000 at Thu 20 Jan 2011 11:09:34
Saved at k=80000 at Thu 20 Jan 2011 11:09:41
A Good Candidate is R(86453)     Shots = 26
Saved at k=90000 at Thu 20 Jan 2011 11:09:48
Saved at k=100000 at Thu 20 Jan 2011 11:09:57
A Good Candidate is R(109297)     Shots = 26
Saved at k=110000 at Thu 20 Jan 2011 11:10:06
Saved at k=120000 at Thu 20 Jan 2011 11:10:15
Saved at k=130000 at Thu 20 Jan 2011 11:10:25
Saved at k=140000 at Thu 20 Jan 2011 11:10:38
Saved at k=150000 at Thu 20 Jan 2011 11:10:50
Saved at k=160000 at Thu 20 Jan 2011 11:11:01
Saved at k=170000 at Thu 20 Jan 2011 11:11:15
Saved at k=180000 at Thu 20 Jan 2011 11:11:30
Saved at k=190000 at Thu 20 Jan 2011 11:11:49
Saved at k=200000 at Thu 20 Jan 2011 11:12:07
Saved at k=210000 at Thu 20 Jan 2011 11:12:28
Saved at k=220000 at Thu 20 Jan 2011 11:12:43
Saved at k=230000 at Thu 20 Jan 2011 11:13:01
Saved at k=240000 at Thu 20 Jan 2011 11:13:19
Saved at k=250000 at Thu 20 Jan 2011 11:13:40
Saved at k=260000 at Thu 20 Jan 2011 11:13:59
Saved at k=270000 at Thu 20 Jan 2011 11:14:19
A Good Candidate is R(270343)     Shots = 26
Saved at k=280000 at Thu 20 Jan 2011 11:14:39
Saved at k=290000 at Thu 20 Jan 2011 11:15:00
Out[196]= $Aborted
```

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